

SET 3: Pure Mathematics

Tutorial 2

1. Change the following from exponential form to logarithmic form:

(a) $3^3 = 27$ [Ans. $\log_3 27 = 3$]

(b) $6^0 = 1$ [Ans. $\log_6 1 = 0$]

(c) $2^{-3} = \frac{1}{8}$ [Ans. $\log_2 \frac{1}{8} = -3$]

(d) $3^2 = 9$ [Ans. $\log_3 9 = 2$]

(e) $\sqrt[5]{32} = 2$ [Ans. $\log_{32} 2 = \frac{1}{5}$]

(f) $\left(\frac{3}{5}\right)^{-2} = \frac{25}{9}$ [Ans. $\log_{3/5} \frac{25}{9} = -2$]

(g) $\sqrt{25} = 5$ [Ans. $\log_{25} 5 = \frac{1}{2}$]

(h) $4^{3/2} = 8$ [Ans. $\log_4 8 = \frac{3}{2}$]

2. Change the following from logarithmic form to exponential form:

(a) $\log_{10} \frac{1}{1000} = -3$ [Ans. $10^{-3} = \frac{1}{1000}$]

(b) $\log_5 625 = 4$ [Ans. $5^4 = 625$]

(c) $\log_{3/2} \left(\frac{4}{9}\right) = -2$ [Ans. $\left(\frac{3}{2}\right)^{-2} = \frac{4}{9}$]

(d) $\log_7 7 = 1$ [Ans. $7^1 = 7$]

(e) $\log_{36/25} \left(\frac{6}{5}\right) = \frac{1}{2}$ [Ans. $\left(\frac{36}{25}\right)^{1/2} = \frac{6}{5}$]

(f) $\log_{27} 9 = \left(\frac{2}{3}\right)$ [Ans. $27^{2/3} = 9$]

(g) $\log 1 = 0$ [Ans. $10^0 = 1$]

(h) $\ln 1 = 0$ [Ans. $e^0 = 1$]

3. Evaluate the following expressions correct to 4 significant figures:

(a) $\frac{4 \log 23.17 - 4 \ln 31.52}{5 \log 23.46}$ [Ans. -1.218]

(b) $7 \left(\frac{1 - e^4}{1 + e^2} \right)$ [Ans. -44.72]

(c) $\frac{\ln(\log 23.7) - e^2}{\ln 45.6 - \log 21.5}$ [Ans. -2.843]

4. Solve the following equations:

(a) $13^{2x-1} = 8^{x+2}$ [Ans. 2.204]

(b) $7(3)^{x-3} = 5^{3x+1}$ [Ans. -0.793]

(c) $\log(2x^2 + 4x - 3) = \log(x^2 + 2x + 12)$ [Ans. -5 and 3]

(d) $\log_5(m-7) - \log_5(m+1) = 1$ [Ans. -3]

(e) $\ln e^{2x-9} = -7$ [Ans. 1]

(f) $8^{\log_8 5} = 3x$ [Ans. $\frac{5}{3}$]

5. Express each of the following expressions as a single logarithm:

(a) $\frac{1}{4}\log_b 81x^4 - \frac{1}{5}\log_b 32z^{10}$ [Ans. $\log_b \frac{3x}{2z^2}$]

(b) $5\log_b r - 3\log_b 2qr - \log_b 4q$ [Ans. $\log_b \frac{r^2}{32q^4}$]

(c) $2\log_b a^3 + 3\log_b ayz + 4\log_b az - 3\log_b aw$ [Ans. $\log_b \frac{a^{10}y^3z^7}{w^3}$]

(d) $\log_b(m^2 + mn + n^2) + \log_b(m - n)$ [Ans. $\log_b(m^3 - n^3)$]

6. In 2010, the population of a country was about 4.5 million and the exponential growth rate was 2.2% per year.

(a) Find the exponential growth function. [Ans. $P(t) = 4.5e^{0.022t}$]

(b) Estimate the population in 2015. [Ans. 5.023]

(c) After how long will the population be 7 million? [Ans. 20.1 years]

(d) After how long will the population be three times that of year 2010? [Ans. 49.9 years]

7. After 500 years, a sample of radium-226 has decayed to 80.4% of its original mass. Find the half-life of radium-226. [Ans. 1590 years]