SET 2

Chapter 5

Quadratic Equations and Inequalities in One Variable

المعادلات و المتباينات التربيعية بمتغير واحد

5.1 Quadratic Equations المعادلات التربيعية

- An equation that contains one variable is said to be a quadratic equation if the highest power of the variable is 2.
- This type of equations can be solved:

  (i) By factorisation بالتحليل (بالتجربة)
  (ii) By using the quadratic formula بالقانون العام
  (iii) By completing the square بإكمال المربع
  (iv) Graphically بالرسم البياني

The general form of quadratic equation is:

$$ax^2 + bx + c = 0$$

Where a, b and c are constants.

5.2 Solving Quadratic Equations by Factorisation حل المعادلات التربيعية بالتحليل

Example 1. Solve the equations: (a) $x^2+2x-8=0$ and (b) $3x^2-11x-4=0$ by factorisation.

Solution:

(a) $x^2+2x-8=0$

$x^2+2x-8=(x+4)(x-2)$

The quadratic equation $x^2+2x-8=0$ becomes:

$(x+4)(x-2)=0$

Either $(x+4)=0 \Rightarrow x=-4$

or $(x-2)=0 \Rightarrow x=2$

So the roots of $\mathbf{x^2+2x-8=0}$ are $\mathbf{x=-4}$ and **2**

(b) $3x^2-11x-4=0$

$3x^2-11x-4=(3x+1)(x-4)$

The quadratic equation $3x^2-11x-4=0$ becomes:

$(3x+1)(x-4)=0$

Either $(3x+1)=0 \Rightarrow x=-\frac{1}{3}$

or $(x-4)=0 \Rightarrow x=4$

So the roots of $\mathbf{3x^2-11x-4=0}$ are $\mathbf{x=-\frac{1}{3}}$ and **4**

5.3 Solving Quadratic Equations by Using the Quadratic Formula

حل المعادلات التربيعية بالقانون العام

- Using quadratic formula is a straightforward method for solving any quadratic equation.
- The quadratic formula that is used to solve a quadratic equation having the form of $ax^2+bx+c=0$ is:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example 2: Solve $x^2+2x-8=0$ by using the quadratic formula.

Solution:

Comparing $x^2+2x-8=0$ with $ax^2+bx+c=0$ gives:

a = 1, b = 2 and c = – 8.

Substituting these values into the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ gives:

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-8)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+32}}{2}=\frac{-2\pm\sqrt{36}}{2}=\frac{-2\pm 6}{2}$$

$$x=\frac{-2+6}{2} \quad \text{or} \quad \frac{-2-6}{2}$$

Therefore $\boldsymbol{x}=\dfrac{4}{2}=\mathbf{2}$ or $\dfrac{-8}{2}=\mathbf{-4}$ (as in example 1(a)).

Example 3: Solve $4x^2+7x+2=0$ by using the quadratic formula.

Solution:

Comparing $4x^2+7x+2=0$ with $ax^2+bx+c=0$ gives:

a = 4, b = 7 and c = 2.

Substituting these values into the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ gives:

$$x=\frac{-7\pm\sqrt{7^2-4(4)(2)}}{2(4)}$$

$$x=\frac{-7\pm\sqrt{49-32}}{8}=\frac{-7\pm\sqrt{17}}{8}$$

$$x=\frac{-7+\sqrt{17}}{8}=-0.360 \quad \text{or} \quad \frac{-7-\sqrt{17}}{8}=-1.390$$

Hence the roots of $\mathbf{4x^2+7x+2=0}$ are $\boldsymbol{x}=-0.360$ and $\boldsymbol{x}=-1.390$

Example 4: Solve $2.68x^2 - 10.3x - 4.2 = 0$ by using the quadratic formula.

Solution:

$a = 2.68$, $b = -10.3$ and $c = -4.2$.

Substituting the values of a, b and c into the quadratic formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-(-10.3) \pm \sqrt{(-10.3)^2 - 4(2.68)(-4.2)}}{2(2.68)}$$

$$x = \frac{10.3 \pm \sqrt{106.09 + 45.024}}{5.36} = \frac{10.3 \pm \sqrt{151.114}}{5.36} = \frac{10.3 \pm 12.293}{5.36}$$

$$x = \frac{10.3 + 12.293}{5.36} = 4.215 \quad \text{or} \quad x = \frac{10.3 - 12.293}{5.36} = -0.372$$

Thus, the roots of $\mathbf{2.68x^2 - 10.3x - 4.2 = 0}$ are $x = 4.215$ and $x = -0.372$

5.4 Inequalities المتباينـات

- An Inequality is a mathematical sentence that includes two unequal sides.
- For example, the two sides of $3x + 4 > 6$ are unequal, and the symbol > indicates that the left-hand side $(3x + 4)$ is larger than the right-hand side which is 6.
- The following symbols are used to denote inequalities:

Symbol	Meaning	
$>$	Is greater than	أكبـر من
$<$	Is less than	أصـغر من
$\geq$	Is greater than or equal to	أكبـر من أو يسـاوي
$\leq$	Is less than or equal to	أصـغر من أو يسـاوي

5.5 Quadratic Inequalities in One Variable المتباينات التربيعية بمتغير واحد

- Inequalities may contain one or more variables.
- This chapter presents solving quadratic inequalities in one variable which may be carried out graphically or algebraically.

Example 5: Solve $x^2 - 8x + 12 \le 0$. Present the solution in interval form and on a number line.

Solution

$$x^2 - 8x + 12 \le 0$$

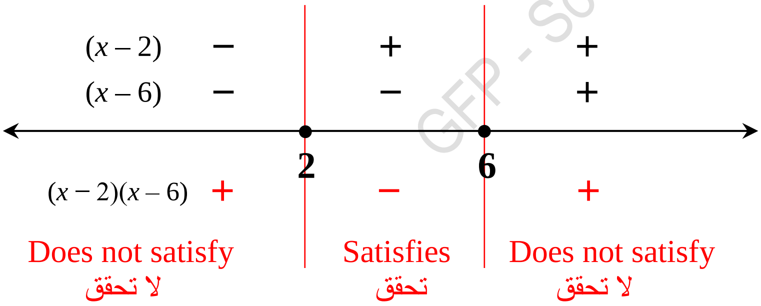
Find the roots of: $x^2 - 8x + 12 = 0$

$$(x-2)(x-6) = 0$$

Either $(x-2) = 0 \Rightarrow x = 2,$

or $(x-6) = 0 \Rightarrow x = 6$

Plot 2 and 6 on a number line and test points to determine the intervals that satisfy the inequality.



- From the previous graph, it is clear that the portion of the number line between 2 and 6 is the solution.
- The critical values 2 and 6 are included in the solution since the inequality is of the $\le$ type and for this reason solid lines and closed circles were drawn at these values of the number line.
- Thus, the solution can be expressed in interval form as $[2, 6]$.
- Alternatively, using number line representation, the solution is as shown below.

Example 6: Solve $-2x^2+5x+3>0$. Present the solution in interval form and use a number line to represent it graphically.

Solution

$$-2x^2+5x+3>0$$

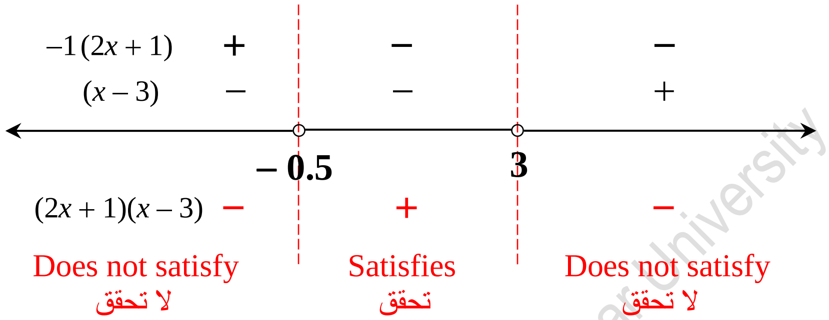
Solve: $$-2x^2+5x+3=0$$

$$-1\,(2x^2-5x-3)=0$$

$$-1\,(2x+1)(x-3)=0$$

Either $(2x+1)=0 \Rightarrow x=-0.5$,

or $(x-3)=0 \Rightarrow x=3$



- The previous figure indicates that the solution is the interval $(-0.5, 3)$.
- The solution can also be represented on a number line and as shown below: