SET 1

Chapter 8

Linear Inequalities in One Variable

المتباينـات الخطيـة بمتغيـر واحـد

8.1 Introduction مقدمة

- Inequalities are mathematical statements involving the symbols $>$, $<$, $\geq$ and $\leq$.
- To solve an inequality means to find a range, or ranges, of values that an unknown x can take and still satisfy the inequality.
- The statement $5x - 4 > 2x + 3$ is an example of inequalities.
- This indicates that the left-hand side, $5x - 4$, is greater than the right-right side, $2x + 3$.
- Four symbols are used to denote inequalities:

$>$	is greater than	أكبر من
$\geq$	is greater than or equal to	أكبر من أو يساوي
$<$	is less than	أصغر من
$\leq$	is less than or equal to	أصغر من أو يساوي

- The arrowhead always points to the smaller side.
- If both sides of an inequality are multiplied or divided by a negative number, the inequality must be reversed.

Example 1. Solve the inequality $x + 3 > 2$ and represent the solution on a number line.

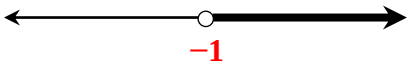
Solution:

$$x + 3 > 2$$

By subtracting 3 from both sides:

$$x + 3 > 2$$
$$x + 3 - 3 > 2 - 3$$
$$x > -1$$

So the solution is $x > -1$.
The solution can be represented on a number line as shown in the figure below.



Example 2. Solve the inequality $3x - 5 \leq 3 - x$ and represent the solution on a number line.

Solution:

$$3x - 5 \leq 3 - x$$

By adding 5 to both sides:

$$3x - 5 + 5 \leq 3 - x + 5$$
$$3x \leq 8 - x$$

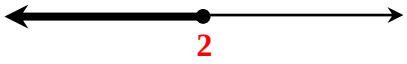
Then adding x to both sides to give:

$$3x + x \leq 8 - x + x$$
$$4x \leq 8$$

Finally dividing both sides by 4 gives:

$$x \leq 2$$

The solution can be represented on a number line as shown in the figure below.



Example 3. Solve the inequality $-2x > 4$ and show the solution on a number line.

Solution:

$$-2x > 4$$

By dividing both sides by -2:

$$\frac{-2x}{-2} < \frac{4}{-2}$$

Remember that because we are dividing by a negative number we must reverse the inequality.

$$x < -2$$

Another solution:

$$-2x > 4$$

By adding $2x$ to both sides:

$$-2x + 2x > 4 + 2x$$
$$0 > 4 + 2x$$

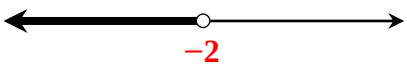
Then, subtracting 4 from both sides gives:

$$0 - 4 > 4 + 2x - 4$$
$$-4 > 2x$$

Finally, dividing both sides by 2 gives:

$$-2 > x$$

Saying that x is less that -2 is the same as saying -2 is greater than x.
The solution is graphically represented in the following figure.



8.2 Inequalities Containing Absolute Value

المتبـاينـات التي تتضمن القيمـة المطلقـة

- Inequalities often includes the absolute value (or modulus) symbol | |.
- For example: $|x| \leq 2$.
- The absolute value of a number is simply its magnitude, regardless of its sign.
- So, $|2| = 2$ and $|-2| = 2$

Example 4. Solve the inequality $|x| \geq 5$ and represent the solution on a number line.

Solution:

$$|x| \geq 5$$

$$x \leq -5 \quad \text{or} \quad x \geq 5$$

This range is shown on the number line below.

Example 5. Solve the inequality $|x - 4| < 3$ and use a number line to represent the solution.

Solution:

$$|x - 4| < 3$$

$$x - 4 < 3 \quad \text{or} \quad x - 4 > -3$$

$$x < 7 \quad \text{or} \quad x > 1$$

This range is shown on the number line below.

Example 6. Solve the inequality $|x - 8| \leq 12$ and use a number line to represent the solution.

Solution:

$$|x - 8| \leq 12$$

$$x - 8 \leq 12 \quad \text{or} \quad x - 8 \geq -12$$

$$x \leq 20 \quad \text{or} \quad x \geq -4$$

The solution is shown on the number line below.

8.3 Real Life Examples مسائل من الواقع

Example 7. The Art Group at Sohar University needs to make a new banner to display during the next theatrical festival. The length of the banner needs to be 5 m. What are the possible widths, if the border of the banner cannot be more than 14 m? Write the inequality and solve it.

Solution:

Let w = the width of the banner.

Then perimeter $= 2(5 + w) = 10 + 2w$

The perimeter should cannot be more than 14 m, thus

$$10 + 2w \leq 14$$

$$2w \leq 14 - 10 \Rightarrow w \leq 2$$

Then, the width should be less than or equal to 2 m.

Example 8. The velocity, in feet per second, of an object fired directly upward is given by $v = 80 - 32t$, where t is the time in seconds. When will the velocity be between 32 and 64 feet per second? Write the inequality, solve it, and interpret it.

Solution:

First, we will set up the double inequality, and then solve for t:

$$32 < 80 - 32t < 64$$

$$32 - 80 < 80 - 80 - 32t < 64 - 80$$

$$-48 < -32t < -16$$

$$\frac{-48}{-32} > \frac{-32t}{-32} > \frac{-16}{-32}$$

$$1.5 > t > 0.5$$

Note that, since we had to divide by a negative number, we had to reverse the inequality signs.

Note also that we might find the above answer to be more easily understood if written the other way around:

$$0.5 < t < 1.5$$

This result may be interpreted as: **The velocity will be between 32 and 64 feet per second between 0.5 seconds and 1.5 seconds after launch.**